NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

APPLYING A FIX-AND-RELAX HEURISTIC TO U.S. NAVY FORCE STRUCTURE PLANNING

by

Mehmet Aytekin

December 2002

Thesis Advisor: Javier Salmeron
Second Reader: Robert F. Dell

Approved for public release; distribution is unlimited

THIS PAGE INTENTIONALLY LEFT BLANK

REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.			
1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** December 2002	**1. AGENCY USE ONLY *(Leave blank)***	
4. TITLE AND SUBTITLE: Applying a Fix-and-Relax Heuristic to U.S. Navy Force Structure Planning			**5. FUNDING NUMBERS**
6. AUTHOR(S) Aytekin, Mehmet			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES) Chief of Naval Operations, Assessment Division (N81)			**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.			
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited			**12b. DISTRIBUTION CODE**

13. ABSTRACT *(maximum 200 words)*

Capital Investment Planning Aid (CIPA) is an optimization-based decision support system created for the U.S. Navy to help plan yearly force structure procurement and retirement. CIPA constraints include yearly industrial and budget limits, as well as mission inventory and force mix requirements. Over a 30-year planning horizon, CIPA helps plan over $1 trillion. Several approaches have been proposed and implemented to solve the CIPA core, a mixed-integer linear program (MILP). Unfortunately, some of these MILPs cannot be solved in a reasonable amount of time using general-purpose commercially available optimization software. This thesis presents a new MILP-based heuristic technique, fix-and-relax, that yields good quality solutions and reduces the computational solution time for our set of realistic test cases.

14. SUBJECT TERMS			**15. NUMBER OF PAGES**
Operations Research, Capital Investment Planning Aid, CIPA, Heuristic, Optimization, Integer Linear Programming			67
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT	**18. SECURITY CLASSIFICATION OF THIS PAGE**	**17. SECURITY CLASSIFICATION OF REPORT**	**18. SECURITY CLASSIFICATION OF THIS PAGE**
Unclassified	Unclassified	Unclassified	Unclassified

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

THIS PAGE INTENTIONALLY LEFT BLANK

APPLYING A FIX-AND-RELAX HEURISTIC TO U.S. NAVY FORCE STRUCTURE PLANNING

Mehmet Aytekin
Lieutenant Junior Grade, Turkish Navy
B.S., Turkish Naval Academy, 1997

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
December 2002

Author: Mehmet Aytekin

Approved by: Javier Salmeron
Thesis Advisor

Robert F. Dell
Second Reader

James N. Eagle, Chairman
Department of Operations Research

THIS PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

Capital Investment Planning Aid (CIPA) is an optimization-based decision support system created for the U.S. Navy to help plan yearly force structure procurement and retirement. CIPA constraints include yearly industrial and budget limits, as well as mission inventory and force mix requirements. Over a 30-year planning horizon, CIPA helps plan over $1 trillion. Several approaches have been proposed and implemented to solve the CIPA core, a mixed-integer linear program (MILP). Unfortunately, some of these MILPs cannot be solved in a reasonable amount of time using general-purpose commercially available optimization software. This thesis presents a new MILP-based heuristic technique, fix-and-relax, that yields good quality solutions and reduces the computational solution time for our set of realistic test cases.

THIS PAGE INTENTIONALLY LEFT BLANK

TABLE OF CONTENTS

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF FIGURES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF TABLES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF SYMBOLS, ACRONYMS AND/OR ABBREVIATIONS

APN	Aircraft Procurement Navy
B&B	Branch-and-Bound
CIPA	Capital Investment Planning Aid
CNO	The Chief of Naval Operations
DoD	Department of Defense
ES	Exact Solver
F&R	Fix-and-Relax
FY	Fiscal Year
GAMS	General Algebraic Modeling System
HS	Heuristic Solver
IWARS	Integrated Warfare Architecture Assessment and Planning Process
LB	Lower Bound
LP	Linear Programming
MILP	Mixed-Integer Linear Program
N51	The Chief of Naval Operations, Strategy and Policy Division
N81	The Chief of Naval Operations, Assessment Division
O&M	Operation and Maintenance
PPBS	Planning, Programming, and Budgeting System
SCIPA	Simplified Model Capital Investment Planning Aid
SCN	Ship Conversion Navy
TOA	Total Obligation Authority
UB	Upper Bound

THIS PAGE INTENTIONALLY LEFT BLANK

ACKNOWLEDGMENTS

I wish to express my deepest gratitude and thanks to my advisor, Professor Javier Salmeron, for his guidance, encouragement, and support throughout my research. His extraordinary efforts in supporting my studies were instrumental to the success of this thesis.

I am also very grateful for the time, effort, and tutoring provided by my second advisor Professor Robert F. Dell.

Finally, I would like to thank my father and mother for giving me encouragement to always take on new challenges.

THIS PAGE INTENTIONALLY LEFT BLANK

EXECUTIVE SUMMARY

Capital Investment Planning Aid (CIPA) is an optimization based decision support system for the U.S. Navy. CIPA prescribes annual force structure procurement and retirement plans based on industrial and budget constraints, as well as mission inventory and force mix requirements over a 30-year horizon.

CIPA's core is a mixed-integer linear problem (MILP). CIPA currently uses two methods for solving the MILP: A heuristic solver (HS) and an exact solver (ES) based on Branch-and-Bound (B&B). The HS is a customized local-search heuristic. The ES is general-purpose commercial solver. Unfortunately, some CIPA instances cannot be solved optimally by the HS, or feasibly with the ES, in reasonable time. This thesis presents a new MILP-based heuristic technique, fix-and-relax (F&R), which yields good quality solutions and reduces the ES computational solution time for our set of realistic test cases.

This thesis also analyzes the "set aside budget" and "set aside labor" concepts implemented in CIPA to diminish the impact of end-effects. The result is a more realistic problem, where procurement levels are better aligned with mission requirements. We show the use of these concepts mitigates over-expenditures in our test cases.

As a result of this research, we recommend using the existing heuristic solver first, along with any exact lower bound. If this does not provide a solution within the desired tolerance, then F&R should be used as a second solver. If F&R cannot find a solution within tolerance, then B&B can be used.

I. INTRODUCTION

"Where are the carriers?" The nation's leaders ask that question whenever a crisis involving vital U.S. interests develops. Not only carriers, but also other naval assets such as destroyers, frigates, mine hunters, landing ships, and submarines are necessary for homeland security. Navy forces have a legitimate role in the execution of national security strategy, but like other military systems can be a target of public and congressional criticism because of their enormous cost [Isenberg, 2002].

The construction, deployment, and use of naval assets have political as well as financial costs. Thus, it is important to determine whether there are less costly ways of planning the procurement and retirement of naval assets to carry out the missions assigned to them. Capital Investment Planning Aid (CIPA) has been developed to help navy analysts plan the retirement and procurement schedules of Navy assets over a 30-year planning horizon. The CIPA core is a mixed-integer linear program (MILP). Unfortunately, some of these MILPs cannot be solved in a reasonable time [Salmeron et al., 2002]. This thesis offers a new approach for solving the existing CIPA MILP that achieves accurate results in most of the existing test cases. The thesis also covers a potential extension of the model to deal with end-effects.

A. U.S. NAVY BUDGET PLANNING

Aircraft carriers are a central part of U.S. defense strategy. The current estimate to develop and build the first CVNX, the next generation U.S. Navy aircraft carrier, is over $10 billion [Scarborough, 2002]. How is this defense budget balanced amongst other investments made by the Department of Defense (DoD)?

DoD uses a Planning, Programming, and Budgeting System (PPBS) to map the best course of action to accomplish its missions. PPBS is a formal, systematic structure for making decisions on policy, strategy, and the development of forces and capabilities to accomplish anticipated missions. PPBS is a cyclic process containing three distinct, but interrelated phases: planning, which produces defense planning guidance; programming, which produces approved program objectives memorandum for the military departments and defense agencies; and budgeting, which produces the DoD portion of the President's national budget [The Chief of Naval Operations (CNO) N6, 2002].

Integrated Warfare Architecture Assessment and Planning Process (IWARS) is part of the U.S. Navy planning process. IWARS comprises five warfare areas (Power Projection, Sea Dominance, Air Dominance, Information Superiority/Sensors, Deterrence) and seven support areas (Sustainment, Infrastructure, Manpower & Personnel, Readiness, Training & Education, Technology, Force Structure), which reflect the complexity of naval warfare requirements and the need to integrate them fully with careful allocation of scarce resources. Each of the 12 IWARS is assessed in an attempt to answer the question of “how much is enough?”, both in terms of quality and quantity, today and in the future [CNO N6, 2002].

B. FORCE STRUCTURE ANALYSIS

CIPA recommends the best yearly force structure procurement plan that satisfies industrial and budget constraints as well as mission inventory and force requirements.

CIPA has two methods to solve its core MILP, a heuristic solver (HS) and an exact solver (ES) [Salmeron et al., 2002].

The HS is a customized local search heuristic that typically returns a plan satisfying the specific requirements in a matter of seconds. The solution accuracy, however, is case-dependent. The HS also yields a valid lower bound, which can be used as an objective assessment of the worst-case quality of the solution returned.

The ES attempts to solve the MILP exactly. CIPA uses The General Algebraic Modeling System (GAMS) [Brooke et al., 1998], a commercial algebraic modeling language to generate the MILP, and solves it using a contemporary commercial solver (e.g., OSL [GAMS-OSL, 2002], CPLEX [GAMS-CPLEX, 2002]). Unfortunately, some CIPA instances cannot be solved optimally in a reasonable time using this general-purpose optimization software. This thesis presents a new MILP-based heuristic technique, fix and relax (F&R), which yields faster answers than ES without compromising the quality of the solution obtained for all test cases considered.

C. STATE OF THE ART

Three theses and one report have been published to date about CIPA.

1. Planning Capital Investments in Navy Forces

Field [1999] presents the first integer-linear program of CIPA. Field tests CIPA using a 25-year planning horizon with eight mission areas, 19 ship classes, five aircraft types, five production facilities, and three categories of money.

2. Optimizing Procurement Planning of Navy Ships and Aircraft

Baran [2000] introduces Generalizing Procurement Planning for Naval Ships and Aircraft (GENSA), which extends the previous version of CIPA. GENSA is tested with a 30-year planning horizon with 29 mission areas, 45 ship classes, 39 aircraft types, 13 production facilities, and four categories of money.

3. Optimized Procurement and Retirement Planning of Navy Ships and Aircraft

Garcia [2001] focuses on improving the underlying optimization modeling for aircraft procurement and retirement scheduling in the Capital Investment Planning Aid with Air Planning Update (CIPA APU). CIPA APU explicitly incorporates the increase in operations and maintenance (O&M) costs of an aircraft by age, and deals with retirement issues by aircraft type and age rather than simply by aircraft type.

4. Capital Investment Planning Aid (CIPA) –an Optimization-Based Decision-Support Tool to Plan Procurement and Retirement of Naval Platforms

Salmeron et al. [2002] describe the planning environment into which CIPA has been introduced, showing how CIPA works, and how CIPA is used. The report presents an overview of CIPA. It describes the planning environment, and presents the latest version of the underlying MILP at the heart of CIPA, discusses exact and heuristic techniques used to solve CIPA, along with their computational performance, and provides an overview of the graphical user interface. Since this report is the latest research about CIPA, it contains the latest version of the MILP used in this thesis.

THIS PAGE INTENTIONALLY LEFT BLANK

II. CIPA MIXED-INTEGER LINEAR PROGRAM

A. MATHEMATICAL MODEL OVERVIEW

CIPA minimizes penalties associated with violating budget constraints, production constraints, and inventory requirements. CIPA gives a recommended plan that includes budget, purchase dates, quantities and cost, and production facility work-force levels. CIPA also isolates force level deficiencies inflicted by budget restrictions on procurements, production that cannot keep pace with procurement requirements, or the lack of any existing replacement for retired platforms. CIPA maintains yearly time resolution for 25 or 30 years. Since it can take up to nine years to build platforms such as destroyers, frigates, and submarines, CIPA's prescriptions for the last few years of the planning horizon may suffer from end effects. The solution for the last years of the horizon may not be accurate because information for years beyond the horizon has not been specified.

The MILP represents a number of features divided into six categories [Salmeron et al., 2002].

1. Mission:
   - Ship-mission and air-mission requirements
2. Inventory:
   - Initial inventory of ships and aircraft
   - Ongoing (resident) production of ships and aircraft
   - Minimum and maximum annual production of ships and aircraft
   - Maximum total production of ships and aircraft
   - Maximum annual inventory of ships and aircraft
   - Minimum and maximum annual ship and aircraft retirement
3. Cost:
   - Ship and aircraft cost profile
   - Economy-of-scale for ship and aircraft procurement
   - Operation and maintenance costs for each ship and aircraft

4. Budget:
   - Minimum and maximum annual budget available
   - Minimum and maximum cumulative budget available
   - Set aside budget (for ships and aircraft)
5. Industry:
   - Work-force profile for ship production
   - Minimum and maximum annual work-force levels for ship industry
   - Set aside labor for ships and its relationship with set aside budget
6. Penalty:
   - Tradeoff among budget shortfall (or surplus), industry work-force shortfall (or surplus) and mission shortfall

Mission requirements (category 1) drive platform procurement. Category 2 features account for yearly platform inventory levels as well as shipyard capacity, minimum retirement levels and the age of existing platforms. Category 3 considers CIPA cost-related features. Procurement costs are typically incurred and spread out over a number of years before a platform is delivered. The cost of purchasing platforms exhibits economies of scale. Category 4 specifies annual and cumulative expenditures that should not exceed or fall below their respective specified limits. Category 5 refers to work-force requirements for ship production that are spread out over the production period of a ship. Ideally, workforce levels should stay within specified limits to prevent the loss of industrial capability and to avoid overtime costs. The last category refers to CIPA penalty charges for each individual violation of budget, industry, or mission-required levels. The penalties express the tradeoff among the different shortfalls and surpluses in order to prioritize the satisfaction of those conditions deemed more critical by the planner.

As main decision variables, the number of platforms procured and retired every year is considered. Additional variables are added to specify the piece-wise linear approximation of non-convex costs associated with economies-of-scale. “Elastic” variables are also incorporated to account for budget, industry, and mission requirement violations. The objective function minimizes the sum of these violations. See Field [1999] for a discussion of how to select penalty values.

All these features are mathematically represented through the following linear program:

$$\begin{aligned} \text{CIPA:} \quad & \min \; F \\ & \text{s.t.} \quad (1) \text{ to } (50) \end{aligned}$$

where the objective function, F, and the constraints (1) and (50), are described in detail in the following section.

B. CIPA MODEL

This section presents the mathematical formulation of the CIPA model presented in Salmeron et al. [2002], page 17, and incorporates the proposed changes for end-effects described on page 119 of the same document. The formulation of the model is included in this document to be comprehensive.

1. Sets and Indices

- Time

Y,	set of years of the planning horizon; $y, y' \in Y$. For convenience, it is assumed that $Y = \{1,2,3\ldots,\lvert Y \rvert\}$

- Platform

A,	set of aircraft types; $a \in A$
S,	set of ship classes; $s \in S$

- Mission

M^A,	set of air missions; $m \in M^A$
M^S,	set of ship missions; $m \in M^S$
$A_m \subseteq A$,	subset of aircraft types that contribute to mission $m \in M^A$
$S_m \subseteq S$,	subset of ship classes that contribute to mission $m \in M^S$

- Production

I_a,	set of cost increments for aircraft $a \in A$; $i \in I_a$
P,	set of production facilities; $p \in P$
$P_s \subseteq P$,	subset of facilities that produce ship class $s \in S$

Q_{spy}, set of quantities available for ship $s \in S$ procurement at facility $p \in P_s$ in year $y \in Y$. This set is defined in terms of the $\underline{sproc}_{spy}$ and $\overline{sproc}_{spy}$ parameters (see below) as follows: $q \in Q_{spy} = \{\underline{sproc}_{spy}, \underline{sproc}_{spy} + 1, \cdots, \overline{sproc}_{spy}\}$

- Others

Z^+, set of non-negative integers, $Z^+ = \{0,1,2,...\}$

2. Parameters (and Units)

- Conventions

The word "procurement" or "to procure" refers to "delivery" or "to deliver", respectively, unless explicitly stated otherwise. Therefore, "procure" is referred to as the action that takes place at the moment (year) that the platform is delivered and available for use from that year onwards, regardless when the real "procurement" arrangements were made.

The words "time period" and "year" are used interchangeably.

The words "facility" and "plant" are used interchangeably.

- Objective-related parameters: Penalties

$ampen_m$, penalty for shortage in completing air mission $m \in M^A$ ($ per aircraft)

$smpen_m$, penalty for shortage in completing ship mission $m \in M^S$ ($ per ship)

$bpen_y^+$, penalty for budget excess ($ per $)

$bpen_y^-$, penalty for budget shortage ($ per $)

$cbpen_y^+$, penalty for cumulative expenses excess ($ per $)

$cbpen_y^-$, penalty for cumulative expenses shortage ($ per $)

$lpen_p^+$, penalty for labor excess at plant $p \in P$ ($ per worker)

$lpen_p^-$, penalty for labor shortage at plant $p \in P$ ($ per worker)

- Constraint-related parameters: used for indices dependencies

SBb_{sp}, number of years before (starting at 0) the procurement of ship class $s \in S$ from plant $p \in P_s$ requires budget (i.e., in 0,1,... $SBb_{sp} - 1$ years before)

SCb_{sp}, number of years before (starting at 0) the procurement of ship class $s \in S$ from plant $p \in P_s$ requires labor (i.e., in 0,1,... $SCb_{sp} - 1$ years before)

SBa_{sp}, number of years after (starting at 1) the procurement of ship class $s \in S$ from plant $p \in P_s$ requires budget (i.e., in 0,1,... SBa_{sp} years before)

SCa_{sp}, number of years after (starting at 1) the procurement of ship class $s \in S$ from plant $p \in P_s$ requires labor (i.e., in 0,1,... SCa_{sp} years before)

ABb_a, number of years before the procurement of aircraft type $a \in A$ in which the aircraft is paid (at once)

- Constraint-related parameters: Ships

$sinv_s^0$, initial inventory of class $s \in S$ ships (number of ships)

$csproc_{sy}$, committed procurement of class $s \in S$ ships in year $y \in Y$ due to production in progress (number of ships)

$\overline{sinv_s}$, maximum number of class $s \in S$ ships in inventory (number of ships)

$\overline{stot_{sp}}$, maximum number of class $s \in S$ ships to procure from plant $p \in P_s$ (number of ships)

$\underline{sproc}_{spy}$, minimum number of class $s \in S$ ships to procure from plant $p \in P_s$ in time period $y \in Y$ (number of ships)

Note: $\underline{sproc}_{spy} = 0,\ \forall s \in S, p \in P_s; \forall y \le \max\{SBb_{sp}, SCb_{sp}\}$ -1 and $\underline{sproc}_{spy} = 0,\ \forall s \in S, p \in P_s; \forall y \ge |Y| + 1 - \max\{SBa_{sp}, SCa_{sp}\}$ is required

$\overline{sproc}_{spy}$, maximum number of class $s \in S$ ships to procure from plant $p \in P_s$ in time period $y \in Y$ (number of ships)

Note: $\overline{sproc}_{spy} = 0,\ \forall s \in S, p \in P_s; \forall y \le \max\{SBb_{sp}, SCb_{sp}\} - 1$ and $\overline{sproc}_{spy} = 0,\ \forall s \in S, p \in P_s; \forall y \ge |Y| + 1 - \max\{SBa_{sp}, SCa_{sp}\}$ is required.

- Constraint-related parameters: Aircraft

$ainv_a^0$, initial inventory of type $a \in A$ aircraft (number of aircraft)

$caproc_{ay}$, committed procurement of type $a \in A$ aircraft in year $y \in Y$ due to production in progress (number of aircraft)

$\overline{ainv}_a$, maximum number of type $a \in A$ aircraft in inventory (number of aircraft)

$\overline{atot}_a$, maximum number of type $a \in A$ aircraft to procure (number of aircraft)

$\underline{aproc}_{ay}$, minimum number of type $a \in A$ aircraft to procure in time period $y \in Y$ (number of ships)

$\overline{aproc}_{ay}$, maximum number of type $a \in A$ aircraft to procure in time period $y \in Y$ (number of ships)

$\underline{inc}_{ayi}$, increment $i \in I_a$ lower bound for the number of type $a \in A$ aircraft to be procured in year $y \in Y$ (number of aircraft)

$\overline{inc}_{ayi}$, increment $i \in I_a$ upper bound for the number of type $a \in A$ aircraft to be procured in year $y \in Y$ (number of aircraft)

$squad_a$, squadron size for aircraft $a \in A$ procurement (number of aircraft)

- Constraint-related parameters: Retirements

$\underline{csret}_{sy}$, minimum cumulative number of class $s \in S$ ships to retire by the end of time period $y \in Y$ (number of ships)

$\overline{csret}_{sy}$, maximum cumulative number of class $s \in S$ ships to retire by the end of time period $y \in Y$ (number of ships)

$\underline{sret}_{sy}$, minimum number of class $s \in S$ ships to retire by the end of time period $y \in Y$ (number of ships)

$\overline{sret}_{sy}$, maximum number of class $s \in S$ ships to retire by the end of time period $y \in Y$ (number of ships)

$\underline{caret}_{ay}$, minimum cumulative number of type $a \in A$ aircraft to retire by the end of time period $y \in Y$ (number of aircraft)

$\overline{caret}_{ay}$, maximum cumulative number of type $a \in A$ aircraft to retire by the end of time period $y \in Y$ (number of aircraft)

$\underline{aret}_{sy}$, minimum number of type $a \in A$ aircraft to retire by the end of time period $y \in Y$ (number of aircraft)

$\overline{aret}_{sy}$, maximum number of type $a \in A$ aircraft to retire by the end of time period $y \in Y$ (number of aircraft)

- Constraint-related parameters: Missions

$seff_{sm}$, effectiveness for ship $s \in S_m$ performing mission $m \in M^S$ (number of missions per ship)

$aeff_{am}$, effectiveness for aircraft $a \in A_m$ performing mission $m \in M^A$ (number of missions per aircraft)

$\underline{smreq}_{my}$, overall effectiveness required for ship mission $m \in M^S$ in time period $y \in Y$ (number of missions)

$\underline{amreq}_{my}$, overall effectiveness required for air mission $m \in M^A$ in time period $y \in Y$ (number of missions)

- Constraint-related parameters: Budget

$oscn_y$, fixed SCN cost in year $y \in Y$ ($)

$ocscn_y$, fixed SCN cost in year $y \in Y$ for ships not considered ($)

$frac$, historical fraction of total SCN cost for ship outfitting

$oapn_y$, fixed APN cost in year $y \in Y$ ($)

$ocapn_y$, fixed APN cost in year $y \in Y$ for aircraft not considered ($)

apn_5, historical fraction of total APN categories 1 through 4 required for categories 5 through 7

oom_y, fixed O&M cost in year $y \in Y$ for maintenance not considered ($)

$scostb_{spql}$, SCN cost incurred l years before q class-s ships are procured from plant p, for $s \in S$, $p \in P_s$, $q \in \bigcup_{y \in Y} Q_{spy}$, $l = \{0, 1, \cdots, SBb_{sp} - 1\}$ ($)

$scosta_{spql}$, SCN cost incurred l years after q class-s ships are procured from plant p, for $s \in S$, $p \in P_s$, $q \in \bigcup_{y \in Y} Q_{spy}$, $l = \{1, \cdots, SBa_{sp}\}$ ($)

$aacost_{ayi}$, increment $i \in I_a$ procurement cost for type $a \in A$ aircraft in year $y \in Y$ ($ per aircraft)

$abcost_{ayi}$, increment $i \in I_a$ fixed procurement cost (intercept) for type $a \in A$ aircraft in year $y \in Y$ (\$)

$omship_{sy}$, O&M cost for class $s \in S$ ship in year $y \in Y$ (\$ per ship)

$omair_{ay}$, O&M cost for type $a \in A$ aircraft in year $y \in Y$ (\$ per ship)

$csbudget_y$, committed budget in year $y \in Y$ due to ship production in progress (\$)

$\underline{toa}_y$, TOA budget lower limit for year $y \in Y$ (\$)

$\overline{toa}_y$, TOA budget upper limit for year $y \in Y$ (\$)

$\underline{ctoa}_y$, TOA cumulative budget lower limit for year $y \in Y$ (\$)

$\overline{ctoa}_y$, TOA cumulative budget upper limit for year $y \in Y$ (\$)

$\overline{ssab}_y$, maximum set aside ship budget for year $y \in Y$ (\$)

$\overline{asab}_y$, maximum set aside aircraft budget for year $y \in Y$ (\$)

- Constraint-related parameters: Labor

$clabor_{py}$, committed labor in year $y \in Y$ at plant $p \in P$ due to production in progress (number of workers)

$sworkb_{spqn}$, required labor n years before q class-s ships are procured from plant p, for $s \in S$, $p \in P_s$, $q \in \bigcup_{y \in Y} Q_{spy}$, $n = \{0,1,\cdots,SCb_{sp}-1\}$ (number of workers)

$sworka_{spqn}$, required labor n years after q class-s ships are procured from plant p, for $s \in S$, $p \in P_s$, $q \in \bigcup_{y \in Y} Q_{spy}$, $n = \{1,\cdots,SCa_{sp}\}$ (number of workers)

$\underline{pcap}_{py}$, minimum production capacity at plant $p \in P$ in time period $y \in Y$ (number of workers)

$\overline{pcap}_{py}$, maximum production capacity at plant $p \in P$ in time period $y \in Y$ (number of workers)

$\overline{sal}_{py}$, maximum set aside labor at plant $p \in P$ in time period $y \in Y$ (number of workers)

$\overline{lcrate}_p$, approximate labor cost at plant $p \in P$ for set aside labor purposes (\$/worker)

3. Decision Variables (and Units)

- Variables related to objective function and to elastic constraints

F, objective function value

α_{my}^{AM}, air mission $m \in M^A$ shortage in year $y \in Y$ (number of aircraft)

α_{my}^{SM}, ship mission $m \in M^S$ shortage in year $y \in Y$ (number of ships)

α_{y}^{B+}, budget excess in year $y \in Y$ (\$)

α_{y}^{B-}, budget shortage in year $y \in Y$ (\$)

α_{y}^{CB+}, cumulative budget excess in year $y \in Y$ (\$)

α_{y}^{CB-}, cumulative budget shortage in year $y \in Y$ (\$)

α_{y}^{L+}, labor excess in year $y \in Y$ (number of workers)

α_{y}^{L-}, labor shortage in year $y \in Y$ (number of workers)

- Main decision variables

$APROC_{ayi}$, number of type $a \in A$ aircraft to procure at the start of year $y \in Y$ in cost increment $i \in I_a$ (number of aircraft)

$ARET_{ay}$, number of type $a \in A$ aircraft to retire by the end of year $y \in Y$ (number of aircraft)

$SPROC_{spyq}$, one if facility $p \in P$ is to deliver $q \in Q_{spy}$ class $s \in S$ ships at the start of year $y \in Y$, and zero otherwise (0-1 variable)

$SRET_{sy}$, number of class $s \in S$ ships to retire by the end of year $y \in Y$ (number of ships)

$SSABudget_y$, amount of budget set aside in year $y \in Y$ for future ship procurements (\$)

$ASABudget_y$, amount of budget set aside in year $y \in Y$ for future aircraft procurements (\$)

$SALabor_{py}$, amount of labor set aside in year $y \in Y$ for future ship procurements from plant $p \in P$ (number of workers)

- Control decision variables

AP_{ayi}, one if aircraft $a \in A$ is procured at the start of year $y \in Y$ in cost increment $i \in I_a$, and zero otherwise (0-1 variable)

$AINV_{ay}$, inventory of type $a \in A$ aircraft at the start of year $y \in Y$ (number of aircraft)

$AMEff_{my}$, overall effectiveness achieved for air mission $m \in M^A$ in year $y \in Y$ (number of missions)

$SINV_{sy}$,	inventory of class $s \in S$ ships at the start of year $y \in Y$ (number of ships)
$SMEff_{my}$,	overall effectiveness achieved for ship mission $m \in M^S$ in year $y \in Y$ (number of missions)
$SBUDGET_y$,	amount of SCN money to budget for year $y \in Y$ (\$)
$ABUDGET_y$,	amount of APN money to budget for year $y \in Y$ (\$)
$OMBUDGET_y$,	amount of O&M money to budget for year $y \in Y$ (\$)
$BUDGET_y$,	total amount of money to budget for year $y \in Y$ (\$)
$LABOR_{py}$,	amount of labor required in year $y \in Y$ at plant $p \in P$ (number of workers)

4. Formulation

$$
\begin{aligned}
\min\ F = &\sum_{y \in Y} \sum_{m \in M^A} ampen_m\, \alpha_{my}^{AM} + \sum_{y \in Y} \sum_{m \in M^S} smpen_m\, \alpha_{my}^{SM} + \\
&\sum_{y \in Y} bpen_y^+\, \alpha_y^{B+} + \sum_{y \in Y} bpen_y^-\, \alpha_y^{B-} + \sum_{y \in Y} cbpen_y^+\, \alpha_y^{CB+} + \sum_{y \in Y} cbpen_y^-\, \alpha_y^{CB-} + \\
&\sum_{y \in Y} \sum_{p \in P} lpen_p^+\, \alpha_{py}^{L+} + \sum_{y \in Y} \sum_{p \in P} lpen_p^-\, \alpha_{py}^{L-}
\end{aligned}
$$

subject to:

- Ship

$$\sum_{q \in Q_{spy}} SPROC_{spyq} = 1, \qquad \forall s \in S, p \in P_s; \forall y \in Y \qquad (1)$$

$$SINV_{sy} = sinv_s^0 + \sum_{y' \in Y | y' \le y} csproc_{sy'} + \sum_{p \in P_s} \sum_{y' \le y} \sum_{q \in Q_{spy'}} q\, SPROC_{spy'q} - \sum_{y' \in Y | y' \le y-1} SRET_{sy'},$$
$$\forall s \in S; \forall y \in Y \qquad (2)$$

$$\sum_{y \in Y} \sum_{q \in Q_{spy}} q\ SPROC_{spyq} \le \overline{stot}_{sp}, \qquad \forall s \in S, p \in P_s \qquad (3)$$

- Aircraft

$$\sum_{i \in I_a} AP_{ayi} = 1, \qquad \forall a \in A; \forall y \in Y \tag{4}$$

$$\underline{inc}_{ayi}\ AP_{ayi} \leq APROC_{ayi} \leq \overline{inc}_{ayi}\ AP_{ayi}, \qquad \forall a \in A, i \in I_a; \forall y \in Y \tag{5}$$

$$\underline{aproc}_{ay} \leq \sum_{i \in I_a} APROC_{ayi} \leq \overline{aproc}_{ay}, \qquad \forall a \in A; \forall y \in Y \tag{6}$$

$$AINV_{ay} = ainv_a^0 + \sum_{y' \in Y | y' \leq y} caproc_{ay'} + \sum_{y' \in Y | y' \leq y} \sum_{i \in I_a} APROC_{ay'i} - \sum_{y' \in Y | y' \leq y-1} ARET_{ay'}, \qquad \forall a \in A; \forall y \in Y \tag{7}$$

$$\sum_{y \in Y} \sum_{i \in I_a} APROC_{ayi} \leq \overline{atot}_a, \qquad \forall a \in A \tag{8}$$

- Retirements

$$\underline{csret}_{sy} \leq \sum_{y' \in Y | y' \leq y} SRET_{sy'} \leq \overline{csret}_{sy}, \qquad \forall s \in S; \forall y \in Y \tag{9}$$

$$\underline{caret}_{ay} \leq \sum_{y' \in Y | y' \leq y} ARET_{ay'} \leq \overline{caret}_{ay}, \qquad \forall a \in A; \forall y \in Y \tag{10}$$

- Mission Inventory

$$SMEff_{my} = \sum_{s \in S_m} seff_{sm}\ SINV_{sy}, \qquad \forall m \in M^s; \forall y \in Y \tag{11}$$

$$SMEff_{my} + \alpha_{my}^{SM} \geq \underline{smreq}_{my}, \qquad \forall m \in M^s; \forall y \in Y \tag{12}$$

$$AMEff_{my} = \sum_{a \in A_m} aeff_{am}\ AINV_{ay}, \qquad \forall m \in M^A; \forall y \in Y \tag{13}$$

$$AMEff_{my} + \alpha_{my}^{AM} \geq \underline{amreq}_{my}, \qquad \forall m \in M^A; \forall y \in Y \tag{14}$$

- Budget

$$SBUDGET_y = oscn_y + (1+frac)\Big(o\csc n_y + csbudget_y + \sum_{s\in S}\sum_{p\in P_s}\sum_{\substack{y'\in Y| \\ y\le y'\le y+SBb_{sp}}}\sum_{q\in Q_{spy'}} s\cos tb_{spq,y'-y}\ SPROC_{spy'q} + \sum_{s\in S}\sum_{p\in P_s}\sum_{\substack{y'\in Y| \\ y-SBa_{sp}\le y'\le y-1}}\sum_{q\in Q_{spy'}} s\cos ta_{spq,y-y'}\ SPROC_{spy'q}\Big), \quad \forall y\in Y \tag{15}$$

$$ABUDGET_y = oapn_y + (1+apn_5)\Big(ocapn_y + \sum_{a\in A}\sum_{i\in I_a}(aa\,cos\,t_{a,y+ABb_a,i}\ APROC_{a,y+ABb_a,i} + ab\,cos\,t_{a,y+ABb_a,i}\ AP_{a,y+ABb_a,i})\Big), \quad \forall y\in Y \tag{16}$$

$$OMBUDGET_y = oom_y + \sum_{s\in S} omship_{sy}\ SINV_{sy} + \sum_{a\in A} omair_{ay}\ AINV_{ay}, \quad \forall y\in Y \tag{17}$$

$$BUDGET_y = SBUDGET_y + ABUDGET_y + OMBUDGET_y + SSABudget_y + ASABudget_y, \quad \forall y\in Y \tag{18}$$

$$\underline{toa}_y \le \alpha_y^{B-} + BUDGET_y, \quad \forall y\in Y \tag{19}$$

$$BUDGET_y - \alpha_y^{B+} \le \overline{toa}_y, \quad \forall y\in Y \tag{20}$$

$$\underline{ctoa}_y \le \alpha_y^{CB-} + \sum_{y'\in Y|y'\le y} BUDGET_{y'}, \quad \forall y\in Y \tag{21}$$

$$\sum_{y'\in Y|y'\le y} BUDGET_{y'} - \alpha_y^{CB+} \le \overline{ctoa}_y, \quad \forall y\in Y \tag{22}$$

- Industrial

$$LABOR_{py} = clabor_{py} + SALabor_{py} + \sum_{s \in S | p \in P_s} \sum_{\substack{y' \in Y | \\ y \le y' \le y + SCb_{sp}}} \sum_{q \in Q_{spy'}} sworkb_{spq,y'-y}\, SPROC_{spy'q} + \sum_{s \in S | p \in P_s} \sum_{\substack{y' \in Y | \\ y - SCa_{sp} \le y' \le y-1}} \sum_{q \in Q_{spy'}} sworka_{spq,y-y'}\, SPROC_{spy'q}, \qquad \forall p \in P; \forall y \in Y \qquad (23)$$

$$\underline{pcap}_{py} \le \alpha^{L-}_{py} + LABOR_{py}, \qquad \forall p \in P; \forall y \in Y \qquad (24)$$

$$LABOR_{py} - \alpha^{L+}_{py} \le \overline{pcap}_{py}, \qquad \forall p \in P; \forall y \in Y \qquad (25)$$

$$\sum_{p \in P} lcrate_p\, SALabor_{py} = SSABudget_y, \qquad \forall p \in P; \forall y \in Y \qquad (26)$$

- Non-negativity and bounds

$$0 \le SSABudget_y \le \overline{ssab}_y, \qquad \forall y \in Y \qquad (27)$$

$$0 \le ASABudget_y \le \overline{asab}_y, \qquad \forall y \in Y \qquad (28)$$

$$0 \le SALabor_{py} \le \overline{sal}_{py}, \qquad \forall p \in P; \forall y \in Y \qquad (29)$$

$$0 \le AINV_{ay} \le \overline{ainv}_a \qquad \forall a \in A; \forall y \in Y \qquad (30)$$

$$AMEff_{my} \ge 0, \qquad \forall m \in M^A; \forall y \in Y \qquad (31)$$

$$0 \le SINV_{sy} \le \overline{sinv}_s, \qquad \forall s \in S; \forall y \in Y \qquad (32)$$

$$SMEff_{my} \ge 0, \qquad \forall m \in M^s; \forall y \in Y \qquad (33)$$

$$\underline{sret}_{sy} \le SRET_{sy} \le \overline{sret}_{sy}, \qquad \forall s \in S; \forall y \in Y \qquad (34)$$

$$\underline{aret}_{ay} \le ARET_{ay} \le \overline{aret}_{ay}, \qquad \forall a \in A; \forall y \in Y \qquad (35)$$

$$SBUDGET_y \ge 0, \qquad \forall y \in Y \qquad (36)$$

$$ABUDGET_y \ge 0, \qquad \forall y \in Y \qquad (37)$$

$$OMBUDGET_y \ge 0, \qquad \forall y \in Y \qquad (38)$$

$$BUDGET_y \ge 0, \qquad \forall y \in Y \qquad (39)$$

$$LABOR_{py} \ge 0, \qquad \forall p \in P; \forall y \in Y \qquad (40)$$

$$\alpha \geq 0 \tag{41}$$

- Fixed variables

$$APROC_{ayi} = 0, \qquad \forall a \in A, i \in I_a; \forall y \in Y \mid y \leq ABb_a \tag{42}$$

$$SPROC_{spy0} = 1, \qquad \forall s \in S, \ p \in P_s; \ \forall y \in Y \mid y \leq \max\{SBb_{sp}, \ SCb_{sp}\} - 1 \tag{43}$$

$$SPROC_{spy0} = 1, \qquad \forall s \in S, p \in P_s; \forall y \in Y \mid y \geq |Y| + 1 - \max\{SBa_{sp}, SCa_{sp}\} \tag{44}$$

- Binary/Integer variables

$$APROC_{ayi} \in Z^+, \qquad \forall a \in A, i \in I_a; \forall y \in Y \tag{45}$$
$$ARET_{ay} \in Z^+, \qquad \forall a \in A; \forall y \in Y \tag{46}$$

$$AP_{ayi} \in \{0,1\}, \qquad \forall a \in A, i \in I_a; \forall y \in Y \tag{47}$$

$$SPROC_{spyq} \in \{0,1\}, \qquad \forall s \in S, p \in P_s; \forall y \in Y; \forall q \in Q_{spy} \tag{48}$$
$$SRET_{sy} \in Z^+, \qquad \forall s \in S; \forall y \in Y \tag{49}$$

An additional constraint requires that:

$APROC_{ayi}$ is a multiple of $squad_a$, $\forall a \in A, i \in I_a; \forall y \in Y$ (50)

Remark: This constraint is not explicitly stated in the formulation. However, notice that it can be easily addressed by setting the proper segment limits. For example, if $squad_a = 4$ then the segment limits could be:

$$\underline{inc}_{ay1} = 0 = \overline{inc}_{ay1}, \underline{inc}_{ay2} = 4 = \overline{inc}_{ay2}, \ \underline{inc}_{ay3} = 8 = \overline{inc}_{ay3}, \ldots$$

Notice that, unless $squad_a = 1$, in which case extra segments are not needed, the number of segments in the model is significantly increased.

5. Description of the Formulation

Specifically, the formulation serves the following purposes:

- The objective function, F, comprises the sum of all the penalties due to Air-Mission and Ship-Mission shortfall, budget deficit and surplus, cumulative budget deficit and surplus, and labor deficit and excess.
- Ship constraints (1) to (3) constrain ship procurement: (1) ensures that one option for ship procurement is executed yearly at each plant, (2) calculates the yearly ship inventory, and (3) limits the maximum procurement from each plant.
- (4) to (8) constrain aircraft procurement: (4) to (6) guarantee that procurements are made within the limits of one specific segment and without exceeding the general minimum and maximum. (7) calculates the yearly aircraft inventory and (8) limits the maximum total procurement throughout the years.
- Cumulative retirement goals are specified in (9) to (10).
- (11) to (14) keep track of platform inventory to perform each specific mission and then calculate mission shortfalls, which depend on the overall effectiveness achieved for each mission.
- Budget constraints (15) to (22) are as follows: (15) calculates the ship-budget per year, which depends on the payment profile for each specific ship that has been procured, (16) is the yearly aircraft budget, considering the segment cost definition, (17) determines O&M costs based on existing inventories. The total yearly budget is assessed in (18), which serves to compute deficits and surpluses on a yearly and cumulative basis in (19) to (22). Notice that the total budget computed in (18) includes the budget set aside to account for end-effects.
- Based on labor profiles for those ships that have been procured, the labor force level required at the different shipyards is estimated in equation (23). Then, the lack of labor or excess is computed in (24) to (25).

- Accounting for end-effects, (26) establishes an approximate relationship between set aside labor and set aside budget.
- (27) to (41) establish non-negativity and bounds for the decision variables. Among these bounds, specified maxima and minima for platform inventory and retirement levels exist, and maximum levels for the set aside budget and set aside labor.
- Some variables need to be fixed in (42) to (44), since they would otherwise involve actions beyond the horizon limits.
- (45) to (49) specify those variables that need to be considered integer or binary. This also implies the integrality of other variables such as platform inventories and mission inventories.
- Finally, (50) requires the aircraft procurement to be a multiple of the squadron size. As the remark indicates, this can be accomplished by adding extra segments for those aircraft whose squadron size for procurement purposes is greater than one.

C. CONSIDERATIONS REGARDING THE CIPA MODEL

The previous work by Field [1999], Baran [2000], and Garcia [2001] did not solve:

$$\begin{aligned}\textbf{CIPA:}\quad & \min\ F \\ & \text{s.t.}\quad \text{(1) to (50)}\end{aligned}$$

but a slightly relaxed version we call "Simplified CIPA (SCIPA)":

$$\textbf{SCIPA:}\quad \min\ F$$

$$s.t.\begin{cases}\text{(1) to (44)} \\ \text{(47) to (48)} \\ APROC_{ayi} \geq 0 \ \text{(45 - modified)} \\ ARET_{ay} \geq 0 \ \text{(46 - modified)} \\ SRET_{sy} \geq 0 \ \text{(49 - modified)}\end{cases}$$

SCIPA relaxes the integrality requirements for aircraft procurement and retirement and for ship retirement. Squadron size requirements for aircraft procurement (50) are also disregarded.

Other constraints such as mission effectiveness or set aside budget and set aside labor constraints are also not considered in previous work by Field [1999], Baran [2000] and Garcia [2001].

SCIPA is used rather than CIPA because it still provides helpful prescriptions and it should be easier to solve. Salmeron et al. [2002] devised a post-processor that heuristically rounds a SCIPA solution and also satisfies the squadron size requirements to produce a feasible solution to the original CIPA MILP.

Consequently, when referring to a solution provided by the exact solver (ES), or simply, to an "exact solution," the solution of the following process is actually being referred to:

Exact Solution= Solve (exactly) SCIPA + Round solution to meet (45)-(46) and (49)-(50).

Of course, the "exact solution" is, in actuality, a heuristic solution that by construction is expected to be relatively close to the optimal.

The thesis does not attempt to improve the solution rounding process. Instead, a technique is devised to efficiently solve SCIPA.

D. SOLVING THE SCIPA MODEL USING FIX-AND-RELAX

In this section we present a general-purpose technique intended to reduce the computational burden of solving SCIPA by B&B. The technique involves using a number of sub-problems each with fewer binary variables than the original MILP. The approach follows Fix-and-Relax (F&R) introduced by Dillenberger et al. [1994]. See also Escudero and Salmeron [2002] and earlier versions of similar techniques used by Brown et al. [1987], among others.

SCIPA can be rewritten as:

$$\textbf{SCIPA:}\ \min f(x,z)$$
$$s.t.\begin{cases} z \in \{0,1\}^n \\ x \geq 0 \\ (x,z) \in XZ \end{cases}$$

where z is a vector that comprises $AP_{ayi}, \forall a \in A,\ i \in I_a; \forall y \in Y$, and $SPROC_{spyq}, \forall s \in S,\ p \in P; \forall y \in Y; \forall q \in Q_{spy}$. These variables are required to be binary, as in the original CIPA, (47) and (48). On the other hand, x is the vector of all the continuous variables of the SCIPA model (APROC, ARET, SRET, α, etc., with their appropriate index domain). $f(x,z) = f(\alpha)$ represents our linear objective function. The set of constraints represented by $(x,z) \in XZ$ is (1) to (44).

To generalize the exposition of the methodology, the components of $\boldsymbol{z}$ are denoted $z_1,...,z_n$, so $\boldsymbol{n}$ is the total number of binary variables in the original model. Let $V = \{1,2,...,n\}$ be the set indices for those variables, and let $V_1,...,V_k$ be a direct partition of the set V, that is, $V_i \subseteq V, \forall i = 1,...,k, V = \bigcup_{i=1}^{k} V_i$, and $V_i \cap V_{i'} = \emptyset, \forall i,i' = 1,...,k \middle| i \neq i'$. The cardinality of each V_i is denoted $|V_i| = n_i$. Therefore, $n = \sum_{i=1,...,k} n_i$, and SCIPA can be rewritten as:

$$\textbf{SCIPA:}\ \min f(x,z)$$
$$\text{s.t.}\begin{cases} z_j \in \{0,1\}, \forall j \in V_i, i = 1,...,k \\ x \geq 0 \\ (x,z) \in XZ \end{cases}$$

In the partition selected for our problem, for a given year y, V_y is defined comprising all the variables of type $AP_{ayi}, \forall a \in A, i \in I_a$ and $SPROC_{spyq}, \forall s \in S, p \in P; \forall q \in Q_{spy}$, i.e., all the variables associated with year y.

The F&R framework solves the following sequence of k mixed-0-1 sub-problems, hereafter called *stages*, and is denoted as $\textbf{SCIPA}^{\text{r}}$, for $r = 1,...,k$. In our approach, $k = |Y|$, but in a more general framework, k would depend on how each stage is defined. $\textbf{SCIPA}^{\text{r}}$ is defined as follows:

$$\textbf{SCIPA}^{\textbf{r}}\textbf{:}\quad \min_{\substack{(x,z)\in XZ\\ x\geq 0}} f(x,z)$$

$$\text{s.t.}\ \begin{cases} z_j = \hat{z}_j, \forall j \in V_i, i = 1,...,r-1 \text{ (if r>1)} \\ z_j \in \{0,1\}, \forall j \in V_r \\ z_j \in [0,1], \forall j \in V_i, i = r+1,...,k \text{ (if r<k)} \end{cases}$$

where the values $\hat{z}_j$ for $j \in V_i, i = 1,...,r-1$ in stage r>1 are retrieved from the solution to problems $\textbf{SCIPA}^{\textbf{1}},...,\textbf{SCIPA}^{\textbf{r-1}}$, respectively. Because only a reduced subset of (non-fixed) 0-1 variables are restricted to be integer at each stage, *r,* we expect each of the $\textbf{SCIPA}^{\textbf{r}}$ models to solve more efficiently than original SCIPA.

In particular, our implementation begins by relaxing the binary constraints for all the variables in z except those associated with period $y = 1$. $\textbf{SCIPA}^{1}$ makes it possible to easily obtain a "what-to-do-first" solution. These binary variables are then fixed at the second stage. In $\textbf{SCIPA}^{2}$, only those variables associated with the second period $(y = 2)$ are deemed integer. This cascade process is followed until the variables for the last period, $y = |Y|$, are set to integer values.

In short, our model (SCIPA) is divided into $k = |Y|$ sub-problems that need to be solved in sequence:

$\mathbf{SCIPA}^{y} : \min f(\alpha)$

$$
s.t.\begin{cases}
(1) \text{ to } (44) \\
APROC_{ayi} \geq 0 \\
ARET_{ay} \geq 0 \\
SRET_{sy} \geq 0 \\
AP_{ay'i} = \hat{AP}_{ay'i}, \forall a \in A, i \in I_a; y' = 1,...,y-1 \text{ (if } y > 1) \\
SPROC_{spy'q} = \hat{SPROC}_{spy'q}, \forall s \in S, p \in P_s; \forall q \in Q_{spy'}; y' = 1,...,y-1 \text{ (if } y > 1) \\
AP_{ayi} \in \{0,1\}, \forall a \in A, i \in I_a \\
SPROC_{spyq} \in \{0,1\}, \forall s \in S, p \in P_s; \forall q \in Q_{spy} \\
AP_{ay'i} \in [0,1], \forall a \in A, i \in I_a; y' = y+1,...,|Y| \text{ (if } y < |Y|) \\
SPROC_{spy'q} \in [0,1], \forall s \in S, p \in P_s; \forall q \in Q_{spy'}; y' = y+1,...,|Y| \text{ (if } y < |Y|)
\end{cases}
$$

If $V^{*}\mathbf{(SCIPA)}$ is allowed to denote the optimal objective function value for our original model, and $\underline{V}\mathbf{(SCIPA)}$ and $\bar{V}\mathbf{(SCIPA)}$ are also allowed to denote a lower bound and an upper bound on that solution, respectively, the F&R algorithm is as follows:

<u>F&R (SCIPA): Fix-and-Relax Algorithm for model SCIPA</u>

Input: Partition $V_1,...,V_k$, where $k = |Y|$, and each V_y contains exactly all the binary variables associated with period *y*:

V_y= {triplets (*a, y, i*) for AP_{ayi} variables} $\cup$ {four-uplas (*s, p, y, q*) for $SPROC_{spyq}$ variables}

Step 1: Set *y*=1 and solve $\mathbf{SCIPA}^{y}$

If $\mathbf{SCIPA}^{y}$ is infeasible, STOP: “Problem SCIPA is infeasible”.

Otherwise, set $\underline{V}\mathbf{(SCIPA)} = V^{*}\mathbf{(SCIPA}^{y}\mathbf{)}$.

Step 2: If *y*=*k*, set $\bar{V}\mathbf{(SCIPA)} = V^{*}\mathbf{(SCIPA}^{k}\mathbf{)}$ and STOP: “Problem SCIPA is feasible”.

Otherwise, increase *y* by 1.

Step 3: Solve $\mathbf{SCIPA}^y$.

If $\mathbf{SCIPA}^y$ is infeasible, STOP: "Problem SCIPA status is unknown".

Otherwise, go back to Step 2

Output: SCIPA status ("Infeasible", "Feasible" or "Unknown"). If status is "Feasible", $\underline{V}$**(SCIPA)** and $\bar{V}$**(SCIPA)** are a lower and an upper bound, respectively, on the optimal solution to SCIPA.

As indicated in Step 3, F&R(SCIPA) has the potential to fail. This may occur if (SCIPA^1) is feasible but, at some stage $y > 1$, the associated problem (SCIPA^y) becomes infeasible. In this situation, F&R(SCIPA) is unable to recognize if the infeasibility is due to the fact that: (a) SCIPA is actually integer-infeasible (but continuous-feasible), or (b) (SCIPA) is integer-feasible, but the cascade fixing procedure, which works with estimates of the true optimal values of the variables, makes (SCIPA^y) infeasible.

In our computational experience, the later problem never occurred, but it if did, alternative versions of this algorithm may be implemented that overcome this difficulty (e.g., Escudero and Salmeron [2002]).

Notice also that F&R(SCIPA) yields a relative gap equal to $(\bar{V}(\text{SCIPA}) - \underline{V}(\text{SCIPA}))/\underline{V}(\text{SCIPA})$. Enhancements of the algorithm (e.g., Escudero and Salmeron [2002]) can deal with the situation where this gap is too big. One technique consists of stepping back and grouping multiple stages into a single one. Eventually, if the gap discrepancies continue, the F&R method becomes a single-stage process solving the original SCIPA.

THIS PAGE INTENTIONALLY LEFT BLANK

III. RESULTS

This chapter analyzes the results extracted from multiple runs of CIPA for different test cases.

GAMS [Brooke et al. (1998)] (version 2.0.8.3 with Revision 117 module) incorporating the CPLEX solver [GAMS-CPLEX (2002)] (version 6.6.1) solves the MILP. Computations are from a Dell Computer Precision 340 Pentium-4, 2 GHz desktop computer with 1 GB of random access memory.

Two main data sets are used for testing purposes. The first is from Baran [2000], and called "Baseline scenario #1." The second, provided by N81, is called "Baseline scenario #2."

Baseline scenario #1 consists of 30 aircraft types, 12 different air missions, 13 plants, 45 ship classes, and 17 different ship missions. The original data set is modified slightly to accommodate set-aside budget and labor data.

Baseline scenario #2 consists of 38 aircrafts types, 10 different air missions, 7 plants, 46 ship classes, and 10 different ship missions. This data set has not been used in any previous thesis research.

We create 24 different cases (1_0,…,1_3,…,6_0,…,6_3) from each baseline scenario. Because we use the same notation to represent the same type of excursion from each baseline scenario, results are presented in separate tables for each scenario. In particular, excursions are created as follows (in either scenario): Mission requirements are increased 10%, 25% and are decreased 15% to create cases 1_1, 1_2 and 1_3, respectively. In addition to the mission requirement increment (MRI), the budget is decreased 20% for each case, yielding cases 2_0 through 2_3. An individual yearly budget (IB) option is added to cases in group 1, creating cases 3_0 through 3_3. The same modification is made to group 2 in order to create cases 4_0 through 4_3. Groups 5 (cases 5_0 through 5_3) and 6 (cases 6_0 through 6_3) are created by removing the cumulative budget (CB) option from groups 3 and 4, respectively.

Each run for cases under scenario #1 contains about 14,000 continuous variables and 4,300 discrete variables. Each run for cases under scenario #2 contains about 15,000 continuous variables and 5,100 discrete variables.

In order to assess the efficiency of F&R, results are compared with two other methods: The first method is a branch-and-bound (B&B) [e.g., Wolsey, 1998], as implemented using default settings in GAMS-CPLEX [2002]. In order to be comprehensive, a post-rounding process of the solution to SCIPA is incorporated in order to attain feasibility of the original CIPA model. In essence, the B&B method can be viewed as a F&R instance with one unique stage, i.e., a single partition spanning all the integer variables of the problem. The second method is a customized local-search heuristic [Salmeron et al., 2002] that typically finds acceptable solutions quickly.

Tables are presented in a compact way in the Appendix. For simplicity of exposition, in this chapter we divide these tables by the following measures of effectiveness: Upper Bound (UB), Lower Bound (LB) and gap.

A. COMPARING UPPER BOUNDS

Tables 1 and 2 show the results for the two scenarios and the best feasible solution (i.e., the best UB) obtained using the three methods.

Baseline Scenario #1

CASE	IB	CB	MRI	BI	UB(Heur) (1)	UB(B&B) (2)	UB (F&R) (3)	Time for UB(B&B) (sec) (2)(4)	Time for UB(F&R) (sec) (3)(4)
1_0		*			123536	129203	144367	177.83	434.90
1_1		*	10%		730261	753120	751905	254.06	330.78
1_2		*	25%		1844213	1917775	1911800	24.02	367.40
1_3		*	-15%		75639	98967	126558	717.84	797.38
2_0		*		-20%	124127	161590	138476	617.88	565.15
2_1		*	10%	-20%	838422	799877	837392	271.89	464.36
2_2		*	25%	-20%	2240723	2273176	2305869	74.48	541.94
2_3		*	-15%	-20%	76390	100936	124498	438.51	799.33
3_0	*	*			177610	No	181272	1201.14	1419.98
3_1	*	*	10%		786317	861380	812775	172.67	473.60
3_2	*	*	25%		1914442	2001053	2015089	45.22	283.13
3_3	*	*	-15%		117154	No	115137	1200.95	1454.37
4_0	*	*		-20%	144655	200819	185948	1163.34	684.35
4_1	*	*	10%	-20%	880290	859989	929985	717.51	633.40
4_2	*	*	25%	-20%	2300018	2289095	2338256	78.39	492.09
4_3	*	*	-15%	-20%	95100	No	108379	1200.98	1038.11
5_0	*				174085	No	184313	1200.95	1332.08
5_1	*		10%		784407	832010	833147	139.00	510.13
5_2	*		25%		1915798	1977670	2005881	36.01	402.40
5_3	*		-15%		120080	No	105776	1200.95	1621.90
6_0	*			-20%	142908	239287	187633	1205.84	720.35
6_1	*		10%	-20%	851852	853657	784293	104.83	824.42
6_2	*		25%	-20%	2178379	2028560	2097786	36.78	389.13
6_3	*		-15%	-20%	98011	No	112660	1200.91	1031.47

(1) Heuristic CPU time is always about 1 minute.
(2) B&B runs with a 20 minute time limit. "No" indicates the case that B&B cannot obtain a feasible solution within the allotted time.
(3) Each stage of F&R obtains the solution using a 3 minute time limit with the total time for all stages shown.
(4) Includes a (negligible) time to round the solution for SRET, ARET, APROC variables.
IB: Individual Budget; **CB**: Cumulative Budget; **MRI**: Mission Requirement Increment; **BI**: Budget Increment

Table 1. Upper Bound (UB) and Computational Run Time for All Methods and All Excursions from Baseline Scenario #1.

Baseline Scenario #2

CASE	IB	CB	MRI	BI	UB(Heur) (1)	UB(B&B) (2)	UB (F&R) (3)	Time for UB(B&B) (sec) (2)(4)	Time for UB(F&R) (sec) (3)(4)
1_0		*			1526377	No	1538233	2400.00	2087.24
1_1		*	10%		1879666	No	1875393	2400.00	1551.54
1_2		*	25%		2495731	2511077	2490478	391.45	1649.67
1_3		*	-15%		1072505	No	1068038	2400.00	1467.56
2_0		*		-20%	1531306	1542857	1538894	502.43	2220.30
2_1		*	10%	-20%	1884332	1880542	1889328	402.32	1694.45
2_2		*	25%	-20%	2508233	No	2499054	2400.00	1237.33
2_3		*	-15%	-20%	1075544	No	1067096	2400.00	1741.88
3_0	*	*			1536413	1528043	1536010	518.43	1426.44
3_1	*	*	10%		1889014	1899922	1878403	387.60	1109.18
3_2	*	*	25%		2508594	2536668	2495177	517.42	1393.43
3_3	*	*	-15%		1079994	1075390	1067605	499.40	1584.10
4_0	*	*		-20%	1544200	No	1601896	2400.00	1685.10
4_1	*	*	10%	-20%	1895842	No	1884112	2400.00	1316.77
4_2	*	*	25%	-20%	2520067	No	2555419	2400.00	1430.99
4_3	*	*	-15%	-20%	1086630	1084394	1069629	492.29	1494.43
5_0	*				1529508	No	1523776	2400.00	1454.77
5_1	*		10%		1880137	1883598	1878234	632.81	1424.91
5_2	*		25%		2501399	2499434	2497334	383.98	1400.31
5_3	*		-15%		1072848	1074187	1067744	423.76	1375.98
6_0	*			-20%	1544863	No	1536329	2400.00	1357.54
6_1	*		10%	-20%	1896190	1886009	1889655	491.33	1478.48
6_2	*		25%	-20%	2521073	2520460	2506490	583.73	1632.49
6_3	*		-15%	-20%	1087255	No	1072530	2400.00	1376.93

(1) Heuristic CPU time is always about 3 minutes.
(2) B&B runs with a 40 minute time limit. "No" indicates the case that B&B cannot obtain a feasible solution within the allotted time.
(3) Each stage of F&R obtains the solution using a 3 minute time limit with the total time for all stages shown.
(4) Includes a (negligible) time to round the solution for SRET, ARET, APROC variables.
IB: Individual Budget; **CB**: Cumulative Budget; **MRI**: Mission Requirement Increment; **BI**: Budget Increment

Table 2. UB and Computational Run Time for All Methods and All Excursions from Baseline Scenario #2.

There are several cases that cannot be solved with B&B, or even obtain a feasible solution within 20 minutes (scenario #1) and 40 minutes (scenario #2). Moreover, in most of these cases, no solution at all is obtained even if this method is executed for several hours. By employing the F&R methodology, feasible solutions are obtained within, approximately, 10-20 minutes for scenario 1, and 20-40 minutes for scenario #2.

The upper bound provided by the B&B (if any) and F&R methods is compared to that of a customized heuristic for the problem. In most cases under scenario #1, the heuristic bound is superior. For scenario #2, the F&R bound is, in general, the best, although both the heuristic and B&B bounds are very close to that value.

It is verified by inspection that the trade-off between computational time and solution value achieved favors the use of the heuristic method to compute such solution. However, it cannot be ruled out that, for other scenarios or future specifications of the problem, the heuristic will not behave as well as for scenarios #1 and #2 which served as test-cases for its development. In such a situation, and with B&B not being computationally affordable, a general “quasi-exact” approach, such as F&R, may still be needed.

B. COMPARING LOWER BOUNDS

Solving the linear programming (LP) relaxation of CIPA (or SCIPA) takes only about one minute for both scenarios #1 or #2. When using B&B, a better LB can be obtained by inspecting the B&B tree and selecting the best, least cost, active node. For the F&R method, the best LB is provided by the optimal solution to stage 1. These three bounds are illustrated in Tables 3 and 4.

As Tables 3 and 4 show, the three LBs look very similar in all cases, and in fact coincide for the LP relaxation and F&R.

Baseline Scenario #1

CASE	IB	CB	MRI	BI	LP Relaxation	LB(B&B)	LB(F&R)
1_0		*			119578	119628	119578
1_1		*	10%		718213	718359	718213
1_2		*	25%		1774185	1774201	1774185
1_3		*	-15%		72954	73000	72954
2_0		*		-20%	119578	119673	119578
2_1		*	10%	-20%	734973	742579	734973
2_2		*	25%	-20%	2049108	2068374	2049108
2_3		*	-15%	-20%	72954	73000	72954
3_0	*	*			150432	150432	150432
3_1	*	*	10%		750281	751046	750281
3_2	*	*	25%		1828030	1828053	1828030
3_3	*	*	-15%		103747	103747	103747
4_0	*	*		-20%	135443	135566	135443
4_1	*	*	10%	-20%	751577	762405	751577
4_2	*	*	25%	-20%	2070090	2088395	2070090
4_3	*	*	-15%	-20%	88766	88766	88766
5_0	*				150432	150432	150432
5_1	*		10%		750281	750637	750281
5_2	*		25%		1828030	1828056	1828030
5_3	*		-15%		103747	103747	103747
6_0	*			-20%	135443	135566	135443
6_1	*		10%	-20%	741844	744259	741844
6_2	*		25%	-20%	1852129	1855145	1852129
6_3	*		-15%	-20%	88766	88766	88766

IB: Individual Budget; **CB**: Cumulative Budget; **MRI**: Mission Requirement Increment; **BI**: Budget Increment

Table 3. Lower Bound (LB) for Baseline Scenario #1.

Baseline Scenario #2

CASE	IB	CB	MRI	BI	LP Relaxation	LB(B&B)	LB(F&R)
1_0		*			1520923	1520923	1520923
1_1		*	10%		1874149	1874149	1874149
1_2		*	25%		2487142	2487142	2487142
1_3		*	-15%		1066127	1066127	1066127
2_0		*		-20%	1520923	1521029	1520923
2_1		*	10%	-20%	1874149	1874149	1874149
2_2		*	25%	-20%	2487142	2487142	2487142
2_3		*	-15%	-20%	1066127	1066127	1066127
3_0	*	*			1520923	1520923	1520923
3_1	*	*	10%		1874149	1874149	1874149
3_2	*	*	25%		2487142	2487193	2487142
3_3	*	*	-15%		1066127	1066127	1066127
4_0	*	*		-20%	1520923	1520923	1520923
4_1	*	*	10%	-20%	1874149	1874149	1874149
4_2	*	*	25%	-20%	2487142	2487142	2487142
4_3	*	*	-15%	-20%	1066127	1066127	1066127
5_0	*				1520923	1520923	1520923
5_1	*		10%		1874149	1874149	1874149
5_2	*		25%		2487142	2487143	2487142
5_3	*		-15%		1066127	1066127	1066127
6_0	*			-20%	1520923	1520923	1520923
6_1	*		10%	-20%	1874149	1874149	1874149
6_2	*		25%	-20%	2487142	2487142	2487142
6_3	*		-15%	-20%	1066127	1066127	1066127

IB: Individual Budget; **CB**: Cumulative Budget; **MRI**: Mission Requirement Increment; **BI**: Budget Increment

Table 4. LB for Baseline Scenario #2.

C. GAP COMPARISON

In order to assess the quality of the solution obtained, we define a relative gap for each method as the ratio $\frac{UB-LB}{LB}$, where UB and LB refer to the upper and lower bounds, respectively, provided by the method. For the heuristic method, we take the LP relaxation as a lower bound.

Overall, the heuristic methodology yields the best solution among the three methods for scenario #1 (see Table 5) and provides the solution within acceptable tolerance for scenario #2 (see Table 6). This is not surprising since the heuristic solver was developed and tested using these scenarios as training cases.

As shown on Table 6 for base line scenario #2, F&R is more accurate than the other methods. In particular, B&B cannot reach a feasible solution within a given time in many instances of this scenario. Although F&R is the overall winner for scenario #2, the heuristic solution is as almost the same quality as the F&R solution.

Baseline Scenario #1

CASE	IB	CB	MRI	BI	Gap (Heur)	Gap (B&B) (1)	Gap (F&R)
1_0		*			3.31%	8.00%	20.73%
1_1		*	10%		1.68%	4.84%	4.69%
1_2		*	25%		3.95%	8.09%	7.76%
1_3		*	-15%		3.68%	35.57%	73.48%
2_0		*		-20%	3.80%	35.03%	15.80%
2_1		*	10%	-20%	14.08%	7.72%	13.94%
2_2		*	25%	-20%	9.35%	9.90%	12.53%
2_3		*	-15%	-20%	4.71%	38.27%	70.65%
3_0	*	*			18.07%	?	20.50%
3_1	*	*	10%		4.80%	14.69%	8.33%
3_2	*	*	25%		4.73%	9.46%	10.23%
3_3	*	*	-15%		12.92%	?	10.98%
4_0	*	*		-20%	6.80%	48.13%	37.29%
4_1	*	*	10%	-20%	17.13%	12.80%	23.74%
4_2	*	*	25%	-20%	11.11%	9.61%	12.95%
4_3	*	*	-15%	-20%	7.14%	?	22.10%
5_0	*				15.72%	?	22.52%
5_1	*		10%		4.55%	10.84%	11.04%
5_2	*		25%		4.80%	8.18%	9.73%
5_3	*		-15%		15.74%	?	1.96%
6_0	*			-20%	5.51%	76.51%	38.53%
6_1	*		10%	-20%	14.83%	14.70%	5.72%
6_2	*		25%	-20%	17.61%	9.35%	13.26%
6_3	*		-15%	-20%	10.42%	?	26.92%

(1) The MIP to SCIPA solution is within 10% tolerance. However after rounding for the SRET, ARET, APROC variables the gap may increase to more than 10%.
IB: Individual Budget; **CB:** Cumulative Budget; **MRI**: Mission Requirement Increment; **BI:** Budget Increment

Table 5. Gap between LB and UB for Baseline Scenario #1.

The unexpected results in Tables 5 and 6 are the cases in which the gap is bigger than 10% for F&R. We next discuss the reason for these large gaps by observing the objective value of case 1_3 in base line scenario #1 throughout the evolution of the F&R algorithm.

Baseline Scenario #2

CASE	IB	CB	MRI	BI	Gap (Heur)	Gap (B&B) (1)	Gap (F&R)
1_0		*			0.36%	?	1.14%
1_1		*	10%		0.29%	?	0.07%
1_2		*	25%		0.35%	0.96%	0.13%
1_3		*	-15%		0.60%	?	0.18%
2_0		*		-20%	0.68%	1.44%	1.18%
2_1		*	10%	-20%	0.54%	0.34%	0.81%
2_2		*	25%	-20%	0.85%	?	0.48%
2_3		*	-15%	-20%	0.88%	?	0.09%
3_0	*	*			1.02%	0.47%	0.99%
3_1	*	*	10%		0.79%	1.38%	0.23%
3_2	*	*	25%		0.86%	1.99%	0.32%
3_3	*	*	-15%		1.30%	0.87%	0.14%
4_0	*	*		-20%	1.53%	?	5.32%
4_1	*	*	10%	-20%	1.16%	?	0.53%
4_2	*	*	25%	-20%	1.32%	?	2.75%
4_3	*	*	-15%	-20%	1.92%	1.71%	0.33%
5_0	*				0.56%	?	0.19%
5_1	*		10%		0.32%	0.50%	0.22%
5_2	*		25%		0.57%	0.49%	0.41%
5_3	*		-15%		0.63%	0.76%	0.15%
6_0	*			-20%	1.57%	?	1.01%
6_1	*		10%	-20%	1.18%	0.63%	0.83%
6_2	*		25%	-20%	1.36%	1.34%	0.78%
6_3	*		-15%	-20%	1.98%	?	0.60%

(1) The MIP to SCIPA solution is within 10% tolerance. However after rounding for the SRET, ARET, APROC variables the gap may increase to more than 10%.
IB: Individual Budget; **CB:** Cumulative Budget; **MRI**: Mission Requirement Increment; **BI:** Budget Increment

Table 6. Gap between LB and UB for Baseline Scenario #2.

As Figure 1 and Table 7 show, the gap for the F&R solutions is within acceptable limits. The gap increases suddenly in the post-rounding process, which is used to attain integer feasibility for the original CIPA model. So, the F&R procedure we have developed is actually near-optimal if we consider the model that it has been applied to: SCIPA. The gap for the final CIPA can attributed to the rounding process.

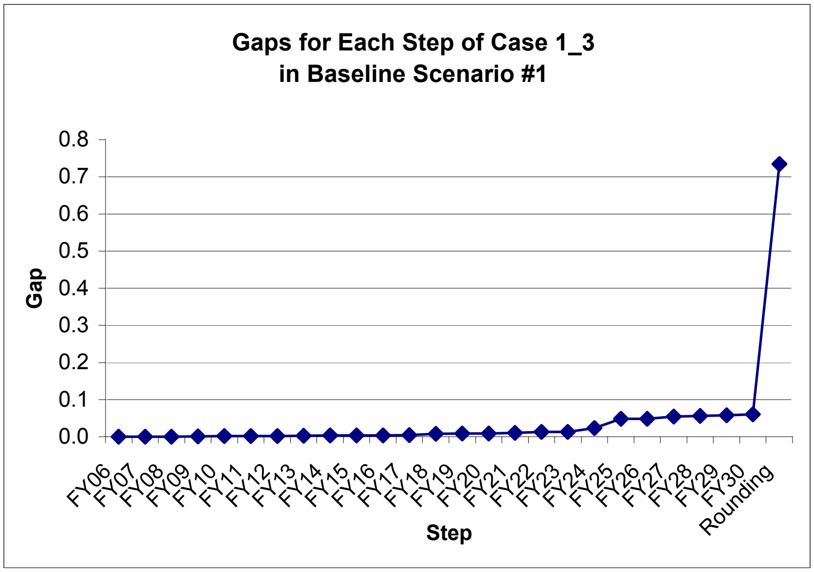

Figure 1. Stage-by-Stage F&R Gap for Case 1_3 in Scenario #1.

Case 1_3 in Baseline Scenario #1

F&R			
Stage	**Obj.Value**	**Gap**	**Time(sec)**
FY06	72954	0.00%	7.72
FY07	72954	0.00%	30.06
FY08	72954	0.00%	23.91
FY09	73015	0.08%	41.89
FY10	73066	0.15%	20.69
FY11	73107	0.21%	43.53
FY12	73107	0.21%	55.17
FY13	73125	0.23%	21.22
FY14	73214	0.36%	47.81
FY15	73214	0.36%	58.19
FY16	73214	0.36%	41.47
FY17	73265	0.43%	49.62
FY18	73539	0.80%	23.59
FY19	73590	0.87%	37.39
FY20	73590	0.87%	33.81
FY21	73684	1.00%	26.31
FY22	73920	1.33%	52.59
FY23	73920	1.33%	17.33
FY24	74639	2.31%	39.33
FY25	76464	4.81%	17.78
FY26	76475	4.83%	34.94
FY27	76929	5.45%	33.27
FY28	77052	5.62%	15.17
FY29	77155	5.76%	9.33
FY30	77374	6.06%	12.06
Rounding	126558	73.48%	3.20
		Total	797.38

B&B		
Stage	**Obj.Value**	**Gap**
B&B	75618	3.59%
Rounding	98967	35.57%

Table 7. Stage-by-Stage F&R Gap and B&B Gap for Case 1_3 in Scenario #1.

Table 7 raises one more interesting issue. When F&R solves one stage, it can yield an integer solution for some subsequent stages. This suggests that part of the time to solve the problem for these stages can be saved. For instance, the objective function value for FY06 does not change for the next two steps. If, after solving FY06, the solution for variables indexed by FY07 and FY08 is verified to already be integer, the solver can continue to find a feasible solution by skipping these stages. This is also valid for FY11-12, FY14-16, FY19-20, and FY22-23. If the strategy for the case shown in Table 7 had been implemented, the total run time would become 537.44 seconds instead of 797.38 seconds.

THIS PAGE INTENTIONALLY LEFT BLANK

IV. END-EFFECTS ANALYSIS

Working with a finite horizon is a simplification driven by problem complexity and limited knowledge of data. Unfortunately, in many cases, using an artificial finite horizon adversely influences the optimal decisions, referred to as end-effects [e.g., Walker et al., 1995]. CIPA optimizes the problem over a 30-year planning horizon, but the actual procurement and retirement of U.S. Navy assets should extend beyond this point. End-effects that emerge because of the mentioned situation arise especially due to two reasons: (1) because no future missions are visualized after the last year, and (2) because the cost and labor structure of some platforms hinder spending money or labor for deliveries.

To overcome this problem, the concepts of “set aside budget” (for ships and aircraft) and “set aside labor” (for ships) are added into the structure of CIPA. In this context, the planner may specify maximum amounts of these categories to be set aside for (undecided) procurements in years beyond the plan’s scope. The maximum labor to be set aside is specified by plant and year [Salmeron et al., 2002]. Also, a consistent relationship between the set-aside budget and set-aside labor is enforced.

The change has been implemented in the CIPA model presented in Chapter III (Equations (18), (23) and (26)). In this chapter, how the model behaves with and without end effects is explored.

A. OVERALL MISSION EFFECTIVENESS WITH AND WITHOUT SET-ASIDES

CIPA aims to minimize budget, industry, and mission requirement violations. One way to reduce minimum budget penalties is to spend money procuring platforms without mission requirements.

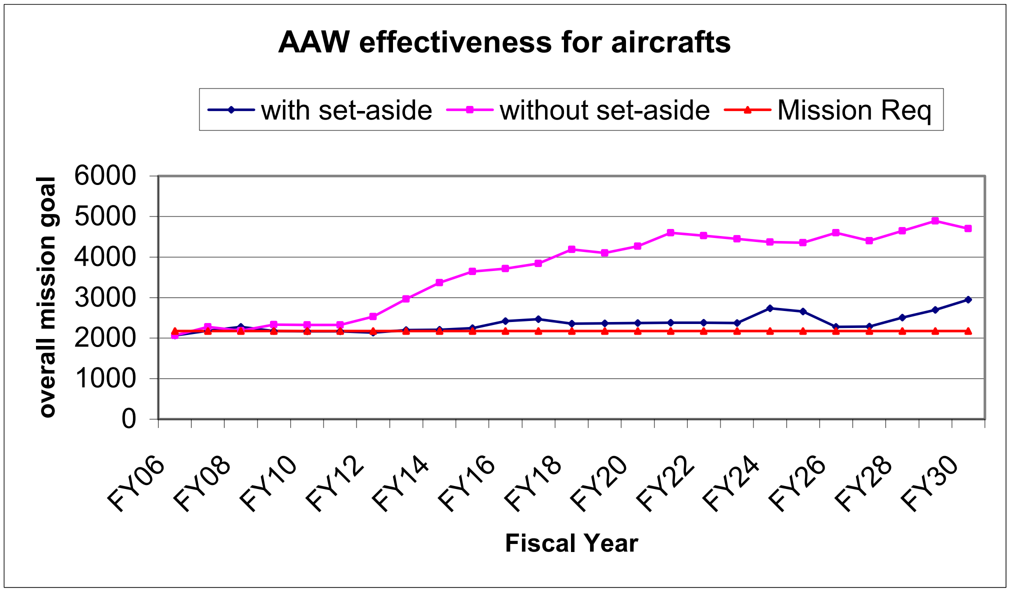

Figure 2. Anti Aircraft Warfare (AAW) Effectiveness for Aircrafts Incorporating Mission Requirements, the Case with and without Set-Aside Concepts.

For example, Figure 2 illustrates that the model without set-aside concepts minimizes the penalties by spending more money to procure and maintain aircraft assets even if they are not needed to satisfy the given mission requirements. In contrast, the model with set-aside concepts follows plausible near mission requirements. This situation is likely if, during some years of the planning horizon, there is a budget surplus mismatch with respect to mission or platform requirements. Allowing set asides to be part of the budget in the last years of the horizon reduces unnecessary expenditures in those years and also in previous years, without violating yearly and cumulative budget limits. (There are other missions where the improvement by using set-aside is not as noticeable as for AAW.)

The impact of end-effects is diminished by using set aside budget and set aside labor. Without this feature, the results may be misled by end-effects. Consequently, we are getting closer to reality by adding the set aside budget and set aside labor concepts to CIPA.

V. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDIES

This thesis shows that F&R is a reliable methodology for solving CIPA MILPs. The structure of this MILP lends itself to a time-based decomposition, which is the approach used for implementing F&R. However, F&R offers a variety of alternatives that may work even better than the one-period-per-stage strategy. This thesis opens a new door for further studies based on our current implementation outcomes. For example, F&R could be improved by grouping variables by ship class, aircraft type or multiple time periods. We may also define a "worth" for each integer decision variable in the problem, associated with the cost of the platform, or its overall effectiveness, or both. Variables would be grouped according to their worth (with the most valuable variables grouped in early stages), and the same F&R methodology can be applied.

Another enhancement to our current implementation is checking variables' status before solving further stages. This would allow us to skip consecutive stages that have all their variables already set to integer values, which in turn decreases the total run time of F&R.

Even though F&R may fail to converge to a solution within 10% tolerance, we demonstrate that in our test cases this gap can be attributed to the rounding process after solving the SCIPA model, rather than the F&R technique itself. This suggests revising the rounding process, or implement F&R directly for CIPA, instead of for SCIPA.

The existing heuristic method gives good results for most of the cases tested in this thesis. However, it should be taken into consideration that these cases were used as training cases to develop the heuristic solver. Thus, we cannot guarantee that this method can always be successfully applied to CIPA.

We analyze the so-called "set aside budget" and "set aside labor" concepts for diminishing end-effects. The incorporation of these concepts into our MILP yields more realistic results by providing effectiveness levels closer to mission requirement levels and, in general, by not over-expending during the planning horizon. Considering the possibility of set-aside budget and set-aside labor allows us to adjust the procurement plan in order to achieve the same level of mission accomplishment avoiding while unnecessary over-expenditures.

As a future extension, we realize that maximum amounts of the set-aside budget and labor might be part of the optimization decisions, rather than input data. Further research may also implement infinite-horizon linear programming. More realistic results may be obtained by using either approach.

As a result of this research, we recommend using the existing heuristic solver first, along with any exact lower bound. If this does not provide a solution within the desired tolerance, then F&R should be used as a second solver. If F&R cannot find a solution within tolerance, then B&B can be used.

It is important to keep focusing on the modeling aspects of the problem to ensure that it meets the U.S. Navy needs as closely as possible, as well as developing efficient solving techniques. We think that techniques using decomposition methods and taking advantage of the problem structure are the most promising ones.

APPENDIX. COMPARISON OF BASE LINE SCENARIOS INCORPORATING WITH HEURISTIC, BRANCH-AND-BOUND (B&B) AND FIX-AND-RELAX (F&R) SOLVERS

Baseline Scenario #1

CASE	B	IB	CB	MRI	BI	LP Relaxation	LB(B&B)	LB(F&R)	UB(Heur) (1)	UB(B&B) (2)	UB (F&R) (3)	Time for LB(B&B) (sec)	Time for UB(B&B) (sec) (2)(4)	Time for UB(F&R) (sec) (3)(4)	Gap (Heur)	Gap (B&B) (5)	Gap (F&R)
1_0	*		*			119578	119628	119578	123536	129203	144367	7.84	177.83	434.90	3.31%	8.00%	20.73%
1_1	*		*	10%		718213	718359	718213	730261	753120	751905	7.61	254.06	330.78	1.68%	4.84%	4.69%
1_2	*		*	25%		1774185	1774201	1774185	1844213	1917775	1911800	5.75	24.02	367.40	3.95%	8.09%	7.76%
1_3	*		*	-15%		72954	73000	72954	75639	98967	126558	9.17	717.84	797.38	3.68%	35.57%	73.48%
2_0	*		*		-20%	119578	119673	119578	124127	161590	138476	6.42	617.88	565.15	3.80%	35.03%	15.80%
2_1	*		*	10%	-20%	734973	742579	734973	838422	799877	837392	8.89	271.89	464.36	14.08%	7.72%	13.94%
2_2	*		*	25%	-20%	2049108	2068374	2049108	2240723	2273176	2305869	9.61	74.48	541.94	9.35%	9.90%	12.53%
2_3	*		*	-15%	-20%	72954	73000	72954	76390	100936	124498	10.41	438.51	799.33	4.71%	38.27%	70.65%
3_0	*	*	*			150432	150432	150432	177610	No	181272	14.72	1201.14	1419.98	18.07%	?	20.50%
3_1	*	*	*	10%		750281	751046	750281	786317	861380	812775	18.36	172.67	473.60	4.80%	14.69%	8.33%
3_2	*	*	*	25%		1828030	1828053	1828030	1914442	2001053	2015089	10.38	45.22	283.13	4.73%	9.46%	10.23%
3_3	*	*	*	-15%		103747	103747	103747	117154	No	115137	13.05	1200.95	1454.37	12.92%	?	10.98%
4_0	*	*	*		-20%	135443	135566	135443	144655	200819	185948	21.86	1163.34	684.35	6.80%	48.13%	37.29%
4_1	*	*	*	10%	-20%	751577	762405	751577	880290	859989	929985	22.27	717.51	633.40	17.13%	12.80%	23.74%
4_2	*	*	*	25%	-20%	2070090	2088395	2070090	2300018	2289095	2338256	15.94	78.39	492.09	11.11%	9.61%	12.95%
4_3	*	*	*	-15%	-20%	88766	88766	88766	95100	No	108379	18.53	1200.98	1038.11	7.14%	?	22.10%
5_0	*	*				150432	150432	150432	174085	No	184313	14.42	1200.95	1332.08	15.72%	?	22.52%
5_1	*	*		10%		750281	750637	750281	784407	832010	833147	12.75	139.00	510.13	4.55%	10.84%	11.04%
5_2	*	*		25%		1828030	1828056	1828030	1915798	1977670	2005881	9.81	36.01	402.40	4.80%	8.18%	9.73%
5_3	*	*		-15%		103747	103747	103747	120080	No	105776	14.94	1200.95	1621.90	15.74%	?	1.96%
6_0	*	*			-20%	135443	135566	135443	142908	239287	187633	13.33	1205.84	720.35	5.51%	76.51%	38.53%
6_1	*	*		10%	-20%	741844	744259	741844	851852	853657	784293	15.53	104.83	824.42	14.83%	14.70%	5.72%
6_2	*	*		25%	-20%	1852129	1855145	1852129	2178379	2028560	2097786	10.50	36.78	389.13	17.61%	9.35%	13.26%
6_3	*	*		-15%	-20%	88766	88766	88766	98011	No	112660	17.89	1200.91	1031.47	10.42%	?	26.92%

(1) Heuristic CPU time is always about 1 minute.
(2) B&B runs with a 20 minute time limit. "No" indicates the case that B&B cannot obtain a feasible solution within the allotted time.
(3) Each stage of F&R obtains the solution using a 3 minute time limit with the total time for all stages shown.
(4) Includes a (negligible) time to round the solution for SRET, ARET, APROC variables.
(5) The MIP to SCIPA solution is within 10% tolerance. However after rounding for the SRET, ARET, APROC variables the gap may increase to more than 10%.
B: Baseline Scenario #1; **IB**: Individual Budget; **CB**: Cumulative Budget; **MRI**: Mission Requirement Increment; **BI**: Budget Increment

Table 8. Comparison of All Methods and All Excursions from Baseline Scenario #1.

Baseline Scenario #2

CASE	B	IB	CB	MRI	BI	LP Relaxation	LB(B&B)	LB(F&R)	UB(Heur) (1)	UB(B&B) (2)	UB (F&R) (3)	Time for LB(B&B) (sec)	Time for UB(B&B) (sec) (2)(4)	Time for UB(F&R) (sec) (3)(4)	Gap (Heur)	Gap (B&B) (5)	Gap (F&R)
1_0	*		*			1520923	1520923	1520923	1526377	No	1538233	8.70	2400.00	2087.24	0.36%	?	1.14%
1_1	*		*	10%		1874149	1874149	1874149	1879666	No	1875393	8.42	2400.00	1551.54	0.29%	?	0.07%
1_2	*		*	25%		2487142	2487142	2487142	2495731	2511077	2490478	12.89	391.45	1649.67	0.35%	0.96%	0.13%
1_3	*		*	-15%		1066127	1066127	1066127	1072505	No	1068038	10.34	2400.00	1467.56	0.60%	?	0.18%
2_0	*		*		-20%	1520923	1521029	1520923	1531306	1542857	1538894	10.31	502.43	2220.30	0.68%	1.44%	1.18%
2_1	*		*	10%	-20%	1874149	1874149	1874149	1884332	1880542	1889328	9.56	402.32	1694.45	0.54%	0.34%	0.81%
2_2	*		*	25%	-20%	2487142	2487142	2487142	2508233	No	2499054	11.30	2400.00	1237.33	0.85%	?	0.48%
2_3	*		*	-15%	-20%	1066127	1066127	1066127	1075544	No	1067096	9.05	2400.00	1741.88	0.88%	?	0.09%
3_0	*	*	*			1520923	1520923	1520923	1536413	1528043	1536010	22.08	518.43	1426.44	1.02%	0.47%	0.99%
3_1	*	*	*	10%		1874149	1874149	1874149	1889014	1899922	1878403	14.20	387.60	1109.18	0.79%	1.38%	0.23%
3_2	*	*	*	25%		2487142	2487193	2487142	2508594	2536668	2495177	14.19	517.42	1393.43	0.86%	1.99%	0.32%
3_3	*	*	*	-15%		1066127	1066127	1066127	1079994	1075390	1067605	10.55	499.40	1584.10	1.30%	0.87%	0.14%
4_0	*	*	*		-20%	1520923	1520923	1520923	1544200	No	1601896	13.06	2400.00	1685.10	1.53%	?	5.32%
4_1	*	*	*	10%	-20%	1874149	1874149	1874149	1895842	No	1884112	13.36	2400.00	1316.77	1.16%	?	0.53%
4_2	*	*	*	25%	-20%	2487142	2487142	2487142	2520067	No	2555419	12.97	2400.00	1430.99	1.32%	?	2.75%
4_3	*	*	*	-15%	-20%	1066127	1066127	1066127	1086630	1084394	1069629	9.55	492.29	1494.43	1.92%	1.71%	0.33%
5_0	*	*				1520923	1520923	1520923	1529508	No	1523776	11.63	2400.00	1454.77	0.56%	?	0.19%
5_1	*	*		10%		1874149	1874149	1874149	1880137	1883598	1878234	10.48	632.81	1424.91	0.32%	0.50%	0.22%
5_2	*	*		25%		2487142	2487143	2487142	2501399	2499434	2497334	16.86	383.98	1400.31	0.57%	0.49%	0.41%
5_3	*	*		-15%		1066127	1066127	1066127	1072848	1074187	1067744	11.67	423.76	1375.98	0.63%	0.76%	0.15%
6_0	*	*			-20%	1520923	1520923	1520923	1544863	No	1536329	10.00	2400.00	1357.54	1.57%	?	1.01%
6_1	*	*		10%	-20%	1874149	1874149	1874149	1896190	1886009	1889655	11.05	491.33	1478.48	1.18%	0.63%	0.83%
6_2	*	*		25%	-20%	2487142	2487142	2487142	2521073	2520460	2506490	13.95	583.73	1632.49	1.36%	1.34%	0.78%
6_3	*	*		-15%	-20%	1066127	1066127	1066127	1087255	No	1072530	12.39	2400.00	1376.93	1.98%	?	0.60%

(1) Heuristic CPU time is always about 3 minutes.
(2) B&B runs with a 40 minute time limit. "No" indicates the case that B&B cannot obtain a feasible solution within the allotted time.
(3) Each stage of F&R obtains the solution using a 3 minute time limit with total time for all stages shown.
(4) Includes a (negligible) time to round the solution for SRET, ARET, APROC variables.
(5) The MIP to SCIPA solution is within 10% tolerance. However after rounding for the SRET, ARET, APROC variables the gap may increase to more than 10%.
B: Baseline Scenario #2; **IB**: Individual Budget; **CB**: Cumulative Budget; **MRI**: Mission Requirement Increment; **BI**: Budget Increment

Table 9. Comparison of All Methods and All Excursions from Baseline Scenario #2.

LIST OF REFERENCES

Baran, N., "Optimizing Procurement Planning of Navy Ships and Aircraft," Master's Thesis, Operations Research Department, Naval Postgraduate School, Monterey, California, December 2000.

Brooke, A., Kendrick, D., Meeraus, A., and Raman, R., "GAMS: A User's Guide," GAMS Development Corporation, Washington, D.C., 1998.

Brown, G., Graves, G. and Ronen, D., "Scheduling Ocean Transportation of Crude Oil," Management Science 33, No. 3, 1987.

CNO N6, CNO N6 Web Site http://cno-n6.hq.navy.mil/N6E/PPBS/default.htm, accessed September, 2002.

Dillenberger, Ch., Escudero, L. F., Wollensak, A., Zhang, W., "On Practical Resource Allocation For Production Planning and Scheduling With Period Overlapping Setups," European Journal of Operational Research, 75, pp. 275-286, 1994.

Escudero, L. F., Salmeron J., "On A Fix-And-Relax Framework For Large-Scale Resource-Constrained Project Scheduling," in review process, 2002. (Currently available as " Working Report I-2002-10 from University Miguel Hernandez, Elche, Spain.)

Field, R. J., "Planning Capital Investments in Navy Forces," Master's Thesis, Operations Research Department, Naval Postgraduate School, Monterey, California, December. 1999.

GAMS-CPLEX, http://www.gams.com/solvers/solvers.htm#CPLEX, accessed September, 2002.

GAMS-OSL, http://www.gams.com/solvers/solvers.htm#OSL, accessed September, 2002.

Garcia, R. M., "Optimized Procurement and Retirement Planning of Navy Ships and Aircraft," Master's Thesis, Operations Research Department, Naval Postgraduate School, Monterey, California, December, 2001.

Isenberg, D., "The Illusion of Power: Aircraft Carriers and U.S. Military Strategy," http://www.cato.org/pubs/pas/pa134.html, accessed October, 2002.

Salmeron, J., Brown, G., Dell, R., Rowe A., "Capital Investment Planning Aid (CIPA) An Optimization-Based Decision-Support Tool to Plan Procurement and Retirement of Naval Platforms," Technical Report, Naval Postgraduate School, Monterey, California, May 2002.

Walker, S. C., Dell, R. F., Quantifying End Effects in Linear and Integer Programs Using Infinite-Horizon Linear Programming," Paper, Operations Research Department, Naval Postgraduate School, Monterey, California, October 1995.

Scarborough, R., "Rumsfeld Comfortable With Carriers Proposal," Washington Times, p. 8, 05 November 2002.

Wolsey, L. A., "Integer Programming," John Wiley, New York, 1988.

INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center
   Ft. Belvoir, Virginia

2. Dudley Knox Library
   Naval Postgraduate School
   Monterey, California

3. Deniz Kuvvetleri Komutanligi
   Kutuphane
   Bakanliklar, Ankara, TURKEY

4. Chief of Naval Operations (N81)
   Navy Department 2000 Navy Pentagon
   Washington, D.C. 20350

5. Deniz Harp Okulu Komutanligi
   Kutuphane
   Tuzla, Istanbul, TURKEY

6. Professor Javier Salmeron, Code OR/Sa
   Department of Operations Research
   Naval Postgraduate School
   Monterey, California

7. Professor Robert F. Dell, Code OR/De
   Department of Operations Research
   Naval Postgraduate School
   Monterey, California

8. Professor Gerald Brown, Code OR/Bw
   Department of Operations Research
   Naval Postgraduate School
   Monterey, California

9. Bilkent Universitesi Kutuphanesi
   Bilkent, Ankara, TURKEY

10. Orta Dogu Teknik Universitesi Kutuphanesi
Balgat, Ankara, TURKEY

11. Bogazici Universitesi Kutuphanesi
Bebek, Istanbul, TURKEY

12. Dz. Utgm. Mehmet Aytekin
Deniz Kuvvetleri Komutanligi
Bakanliklar, Ankara, TURKEY